GRAVITY

The Nature of Gravity

Gravity was known to humans long before Isaac Newton and Einstein. Our earliest ancestors were aware of gravity. They knew that if an object was thrown upward, it would eventually fall downward. They knew that if they threw they had to throw a spear at an upward angle to strike prey at the same level so long as the prey was not in very close proximity to the hunter. There is no known "discoverer" of gravity. However, Newton and Einstein made significant contributions to the understanding of gravity.

Newton's greatest contribution is that he quantified the effects of gravity. He deduced that gravity was directly proportional to the mass of objects and inversely proportional to the distance between them. Mathematically, he determined that the force of gravity could be quantified as follows:

$$F_g = \frac{Gm_1m_2}{r^2}$$

Where

F_g = the force of gravity

G = the Universal Constant.

$$G \cong 6.674x10^{-11}\ \mathrm{N}\left(m/kg\right)^2$$

m_1 and m_2 = mass in kg

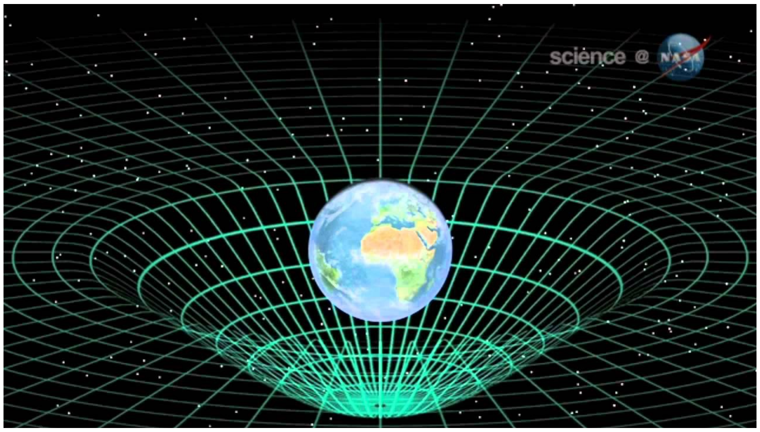
r = radius in meters

Newton also displayed astonishing genius in that he extended the concept of gravity from the earth to the heavens. He determined that gravity produced a force of an object on earth, but he also explained how gravity caused the moon to orbit the earth.

While Newton described and quantified the effects of gravity, he could not explain the nature of gravity.

Albert Einstein took a different approach. He reasoned that no object was truly at rest. A rock laying on the ground is rotating around the axis of the earth; the earth is rotating around the sun, and the sun is rotating around the Milky Way galaxy. He reasoned that all motion was relative. In doing so, he reasoned that for an observer falling freely from the roof of a house there exists no gravitational field.

In 1916, Einstein published his General Theory of Relativity in which he replaced the theory of gravity with geometry, in particular, the curvature of space and time. Einstein's theory of gravity contains equations that calculate how space and time are curved by the presence of matter and energy and how objects move across curved spacetime. This theory described the motion of the moon around the earth as follows; the orbit of the moon is a straight line in spacetime curved by the presence of the sun.



Despite Einstein's conclusions, Newton's equations are so useful that they are used to send satellites to the far reaches of the solar system with incredible accuracy.

Geosynchronous Orbits

When NASA launches a satellite, what keeps the satellite from falling down? This is actually the wrong question. Remember that forces cause **changes** in motion.

An object in orbit experiences a centripetal force and the force due to gravity. If these forces are equal, then there is no change in motion and the motion remains the same; in other words, the satellite remains in orbit.

Centripetal Force:

We have already determined that the acceleration due to circular motion is

$$a = \frac{v^2}{r}$$

and that

$$F = ma$$

Therefore,

$$F_c = \frac{mv^2}{r}$$

We also know that speed is distance per time. Since the circumference of a circle is $2\pi r$

Then the speed of an object travelling in a circular path is defined as

$$v = \frac{2\pi r}{t}$$

As a result, the centripetal force is quantified as follows:

$$F_c = \frac{m}{r}\left(\frac{2\pi r}{t}\right)^2$$

$$F_c = \frac{m}{r}\left(\frac{4\pi^2 r^2}{t^2}\right)$$

$$F_c = \frac{4\pi^2 r}{t^2}$$

Force of Gravity:

Newton defined the force of gravity as follows:

$$F_g = \frac{GM_E m}{r^2}$$

Where,

M_E = Mass of the earth = $5.97x10^{24}kg$

m = Mass of the satellite

G = Gravitational constant = $6.674x10^{-11}$ (S.I. units)

r = Radius from the centre of the earth to the satellite

In order for the satellite travel in an orbit, the centripetal force acting on the satellite must equal the gravitational force acting on the satellite. As a result,

$$\frac{GM_E m}{r^2} = \frac{m4\pi^2 r}{t^2}$$

When we isolate the variable "r", we obtain the following,

$$r = \sqrt[3]{\frac{GM_E t^2}{4\pi^2}}$$

A geosynchronous orbit is one in which the satellite remains at the same point above a geographical location on the earth. Since the earth rotates one revolution every 24 hours, the satellite must also travel a complete circumference in 24 hours.

The earth spins once every 24 hours or once every 86,400 seconds.

When we input the known values into the above equation, we obtain the following,

$$r = \sqrt[3]{\frac{\left(6.674x10^{-11}\right)\left(5.97x10^{24}\right)\left(86400\right)^2}{4\pi^2}} \cong 42,000km$$

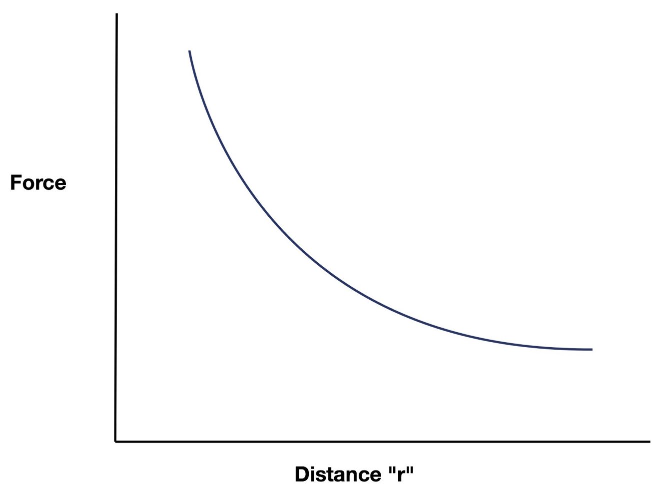
Since r is the radius from the centre of the earth, and since the radius of the earth is approximately 6370 km, the geosynchronous orbit is obtained at an altitude of approximately 36,000 km above the surface of the earth.

This altitude is a prime piece of space real estate because it is where telecommunications and GPS satellites are located.

Potential Gravitational Energy

The strength of the force of gravity follows an inverse square law. The force varies with the inverse square of the distance. (See graph below.)

Note: The masses do not change, so the force varies inversely with the square of the radius between the masses.



We have already determined that energy is the area under the Force - Distance curve (See the Work - Energy Bulletin.)

Calculus shows that the area under the graph (gravitational potential energy - “U”) is defined as follows,

$$U = \frac{GMm}{r}$$

Where “U” is the gravitational potential energy of 2 masses (M,m) a distance (r) apart.

It is important to define signage at this stage. Gravity is an attractive force. This means that when a mass moves closer to a gravitating mass such as earth, work is done on it. Because negative work is done on the object, the associated potential energy is negative. This is analogous to an athlete lifting a barbell. It takes energy to lift the barbell and the athlete will tire. However, the athlete will not tire if the barbell is lowered a certain distance.

Therefore, the total energy of an object moving in the vicinity of a large mass is the kinetic energy minus the gravitational energy. Mathematically, this is expressed as:

$$E = \frac{mv^2}{2} - \frac{GMm}{r}$$

Escape Velocity

Does what goes up necessarily have to come down?

As the distance from the surface of the earth (r) becomes very large and approaches infinity, the area under the force-distance graph approaches zero indicating that the potential gravitational energy approaches zero. As the object approaches the earth, the force on the mass increases and the potential energy of the mass decreases. Also, the mass is barely moving at a large radius (r) and its kinetic energy is also zero. As a result, at a large radius (r), the total energy of the system approaches zero.

The question remains, what speed must a mass attain in order to reach the escape velocity, or the speed at which the mass will not return to earth. If we express this relationship at the surface of the earth, the kinetic energy is the quantity required to throw the mass an infinite distance away. Both the gravitational energy and the potential energy are not zero individually, but they must add up to zero. Mathematically, this relationship is expressed as,

$$0 = \frac{mv^2}{2} - \frac{GM_E n}{R_E}$$

Where

M_E = the mass of the earth

R_E = the radius of the earth

Solving for the velocity (v), we obtain,

$$v = \sqrt{\frac{2GM_E}{R_E}}$$

Solving for the equation, we obtain a velocity of approximately 11 km/s. If the mass attains a speed in excess of this velocity, it will escape the earth's gravitational field and will not return to earth.

As a result, we can conclude that not all that is thrown up, must come down.

Circular Orbit

The total energy of the system can be expressed as,

$$E = \frac{mv^2}{2} - \frac{GMm}{r}$$

However, when and mass is in circular motion such as an object in orbit, the acceleration of the object is defined as,

$$a = \frac{v^2}{r}$$

Since Newton second law is,

$$F = ma$$

Then the centripetal force is defined as,

$$F = \frac{mv^2}{r}$$

For an object to maintain a circular orbit, the centripetal force on the object must equal the gravitational force on the object. As a result,

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$mv^2 = \frac{GMm}{r}$$

$$\frac{mv^2}{2} = \frac{1}{2}\left(\frac{GMm}{r}\right)$$

$$K = \frac{1}{2}U$$

and

$$U = 2K$$

Therefore, in a circular orbit, the kinetic energy is equal to one-half of the potential energy. In such an orbit, the energy of the system can be defined as,

Since

$$E = K - U$$

and since the total energy of the system must be negative because the object is in orbit and the escape velocity has not been reached,

$$E = \frac{1}{2}U = -K$$

This leads us to a counter-intuitive situation. If we increase the kinetic energy of the mass, the total energy goes down! Consider the case of a spacecraft attempting to rendezvous with the International Space Station. The spacecraft is behind the ISS and trying to catch up. If the spacecraft fires its rear engines to accelerate, it would enter a higher orbit and the distance between the craft and the ISS would actually increase. If the craft fired its forward rockets, it would drop to a lower orbit and overtake the ISS. Once in front of the ISS, the spacecraft could then fire its rear thrusters; thereby increasing its orbit and slowing it down to enable a rendezvous. This knowledge will enable you to marvel the next time you see a docking manoeuvre in space.