NEWTON'S LAWS IN 2D & 3D

Review

In the previous bulletins, we discussed Newton's Laws and their interpretation. It is always important to understand the basics before moving into something a little more complex, so let us review the basics once more.

1. Force does not cause motion.
2. Force causes a **change** in motion (magnitude, direction, or both).
3. Do not expect things to move in the direction of the force applied on them. Aristotle was wrong.
4. All circular motion is accelerated motion.

Solving Physics Problems

When confronted with a physics problem, the mind of most individuals starts to race in anticipation of a ready answer. Science is not like that. Science is about knowing the fundamentals well and applying logic in every step of the analysis. In order to help solve a physics problem, the following procedure is recommended.

1. Write $\vec{F} = m\vec{a}$

2. Draw the situation and identify every force acting on the object.

3. Rewrite Newton's second law with all these forces explicitly identified.

4. Establish a Co-ordinate system and rewrite Newton's second law in that co-ordinate system.

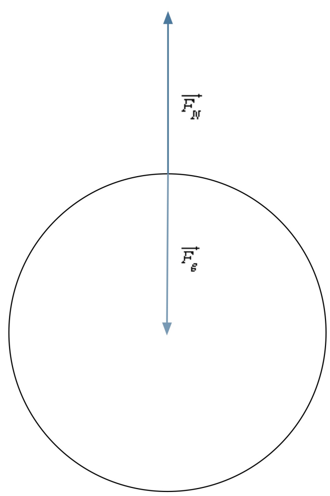
Example #1: Roller Coaster

Question:

You are designing a roller coaster in which there is a loop-the-loop, much like a cork screw, in which the passengers are temporarily upside down. What is the minimum speed required to keep the roller coaster on the rails and the passengers in their seats?

Solution:

$\vec{F} = m\vec{a}$



Where

$\vec{F}_g$ = Force of gravity

$\vec{F}_N$ = The normal force exerted by the roller coaster on the rails.

But, we know that $\vec{F}_g = m\vec{a}$; therefore,

$$mg + \vec{N} = m\vec{a}$$

The minimum force to keep the roller on the track occurs when the normal force of the track on the roller coaster approaches zero. At $\vec{N} = 0$, the roller coaster will start to fall. At this point,

$$m\vec{g} + 0 = m\vec{a}$$

$$m\vec{g} = m\vec{a}$$

We also know that at the very top of the loop, the direction of applied by the force of gravity is in the same direction of the centripetal acceleration (straight down). Therefore, we can alter the equation to deal only with the magnitude of the equation, and not with the directionality. In this case,

$$mg = ma$$

We also know from our review of centripetal force that

$$a = \frac{v^2}{R}$$

Substituting this into the above equation, we obtain

$$mg = \frac{mv^2}{r}$$

When we isolate for velocity, we obtain

$$v = \sqrt{rg}$$

where

r = radius at the top of the roller coaster

g = acceleration of gravity $(9.81\ m/s^2)$

v = limit of minimum velocity in m/s.

This formula represents the velocity of the roller coaster for the track just to touch the track. In order for the roller coaster to stay on the track, the velocity must be over the value obtained by the formula.

Example 2: Vehicle in a Turn With Superelevation

Now let us consider a slightly more complex example with application to collision reconstruction.

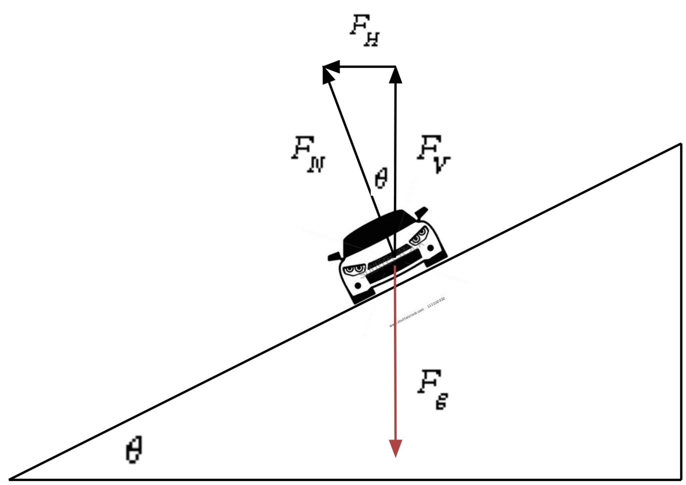
There is a vehicle travelling in a circular path along a frictionless superelevation such as a velodrome made of perfectly smooth ice. What is the optimal angle of the superelevation for any velocity of the vehicle.

Note: If the vehicle goes faster than the minimum, it will glide upwards. If it goes slower, it will slide downwards.

Solution:

$$\vec{F} = m\vec{a}$$

The second step is to draw the situation and identify every force acting on the vehicle.



Where

F_g	=	the force exerted by gravity
F_N	=	the Normal force (90 degrees to the surface of the ramp)
F_H	=	the horizontal component of the force acting on the vehicle
F_V	=	the vertical component the force acting on the vehicle
θ	=	the angle of the superelevation.

The third step of the solution is to rewrite Newton's second law with all these forces explicitly identified.

$$m\vec{g} + \vec{N} = m\vec{a}$$

The fourth step is to identify a co-ordinate system and rewrite Newton's second law in that co-ordinate system.

Let us take the convention that positive is toward the centre of the circle and also up from the surface in the vertical direction.

However, we are now dealing with forces acting along more than one direction. The solution to such problems is to break the forces down into 2 directions along an x and y axis. In this case, the x and y directions represent horizontal and vertical directions.

For now, let us examine the horizontal component of the force.

$$F_H = ma$$

But we know from our examination of centripetal force that

$$a = \frac{v^2}{r}$$

Therefore,

$$F_H = \frac{mv^2}{r}$$

Now, when we look at our force diagram, we recognize that

$$F_H = F_N \sin\theta$$

Therefore,

$$F_N \sin\theta = \frac{mv^2}{r}$$

$$F_N = \frac{mv^2}{r\sin\theta}$$

An examination of our force diagram identifies the following relationship.

$$F_V = F_N \cos\theta$$

Rewriting Newton's second law in the vertical direction with the direction convention as positive for up and negative for down, the following relationship is identified.

$$F_V - F_g = ma$$

Since there is no acceleration in the vertical direction,

a = 0

Therefore,

$$F_V - F_g = 0$$

where

$F_g = mg$ and

$$F_V = F_N \cos\theta$$

Therefore,

$$F_N \cos\theta - mg = 0$$

$$F_N = \frac{mg}{\cos\theta}$$

We have already determined from our analysis of the horizontal component of the force equation that,

$$F_N = \frac{mv^2}{r\sin\theta}$$

Substituting this value into the vertical component of the equation, we obtain the following,

$$\frac{mg}{\cos\theta} = \frac{mv^2}{r\sin\theta}$$

$$\frac{g}{\cos\theta} = \frac{v^2}{r\sin\theta}$$

$$\frac{g\sin\theta}{\cos\theta} = \frac{v^2}{r}$$

$$\frac{\sin\theta}{\cos\theta} = \frac{v^2}{rg}$$

But,

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Therefore,

$$\tan\theta = \frac{v^2}{rg}$$

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

The above formula tells us that, given a known speed, we can determine the angle at which a vehicle can make a turn without slipping up or down.

This formula also has relevance for more than only a vehicle. Think of an airplane making a turn. The aircraft always banks when making a turn. In this manner, it can maintain altitude while in a turn.