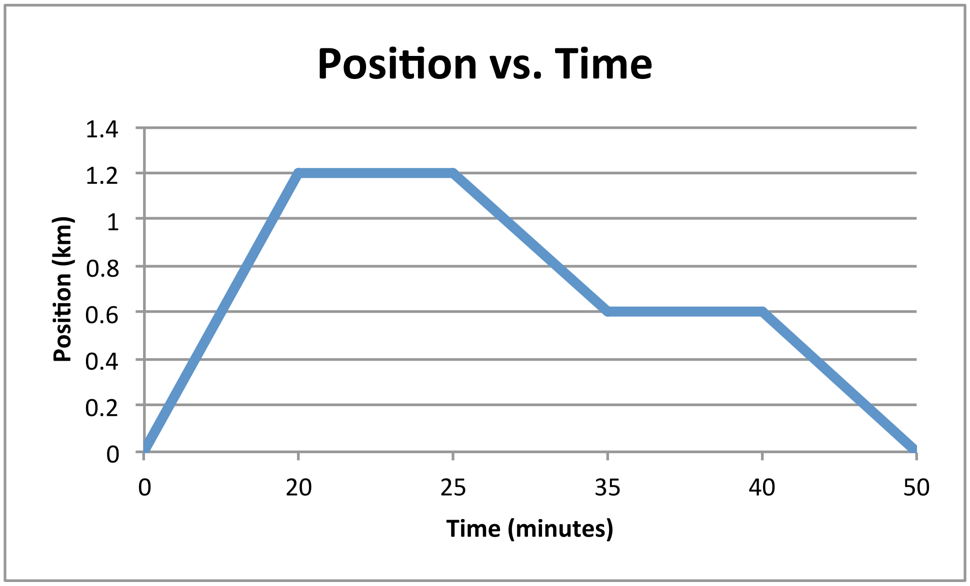
VECTORS

Understanding Vector vs. Scalar Quantities

You will likely have heard scalars defined as quantities of measurement with no directional component and vectors described as quantities of measurement with a directional component. While the definitions appear simple, they are often a source of confusion when applied in a real world environment.

In order to better appreciate the difference, let us consider the following scenario. You start walking from your house to a the convenience store. The store is 1.2 km away, and it takes you 20 minutes to get there. Once at the store, it takes you 5 minutes to locate the items that you want and to pay for them. You then start walking back to your house at the same pace as that you walked to the store. After walking 600 metres, you meet and talk to a friend for 5 minutes before continuing home. The trip from the time that you left the store to the time that you returned home was 25 minutes.

Your position relative to the time taken can be graphed as follows;



The position was measured in kilometres with your home as the reference point. At the start of the walk, you were 0 km from your home. After 20 minutes, you were 1.2 km from your home. After 25 minutes, you were still 1.2 km from your home. After 35 minutes as well at 40 minutes, you were only 0.6 km from your home. Lastly, after 50 minutes, you were back home.

Now let us consider the difference between distance and displacement. In our case, the total distance walked was 2.4 km (1.2 km to the store and 1.2 km return). However, displacement is defined differently. Displacement involves only the difference between the endpoints of the journey. For example, after 40 minutes, you were only 0.6 km from home. The difference between your starting point (home) and your end point after 40 minutes was 0.6 km. It makes no difference that you had actually walked 1.8 km up to that point.

It would also be important to note that your displacement after 50 minutes is zero because you ended up at the same place from which you started.

It may be useful to think of distance as the number on a vehicle's odometer. It does not matter whether or not the vehicle is returning to the same place from which it started. The odometer will always show a positive number that always increases. Displacement, however, is more akin to the distance between 2 points on a map. That distance can increase or decrease.

There is an analogous difference between speed and velocity. Speed is the number that you observe on a vehicle's speedometer. The speedometer does not tell you the direction of travel, nor does it tell you if you are travelling backwards, or returning to your destination.

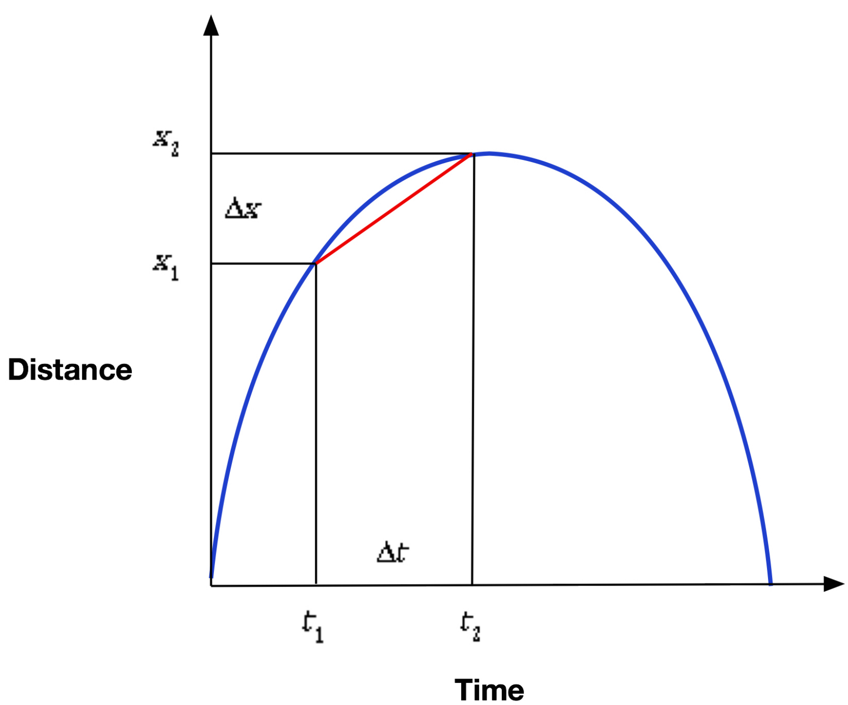
Let us consider the average speed involved in the trip from the store to the point at which you meet your friend and stopped to talk. At that point, you walked 1.8 km (1800 m) over 35 minutes. As a result, your average speed was approximately 51 metres per minute. However, velocity only concerns itself with the end points and does not consider the travel between the endpoints. After 35 minutes, you are only 600 metres from your house. Therefore, your velocity at that time is approximately 17 metres per minutes. This is a significant difference from the average speed measured between the same two points. Velocity involves the rate of change of **<u>displacement</u>** and not the rate of change of distance.

Velocity is the rate of change of displacement (not distance) over time.

Similarly, acceleration involves the rate of change of **<u>velocity</u>** and not the rate of change of speed.

Acceleration is the rate of change of velocity (not speed) over time.

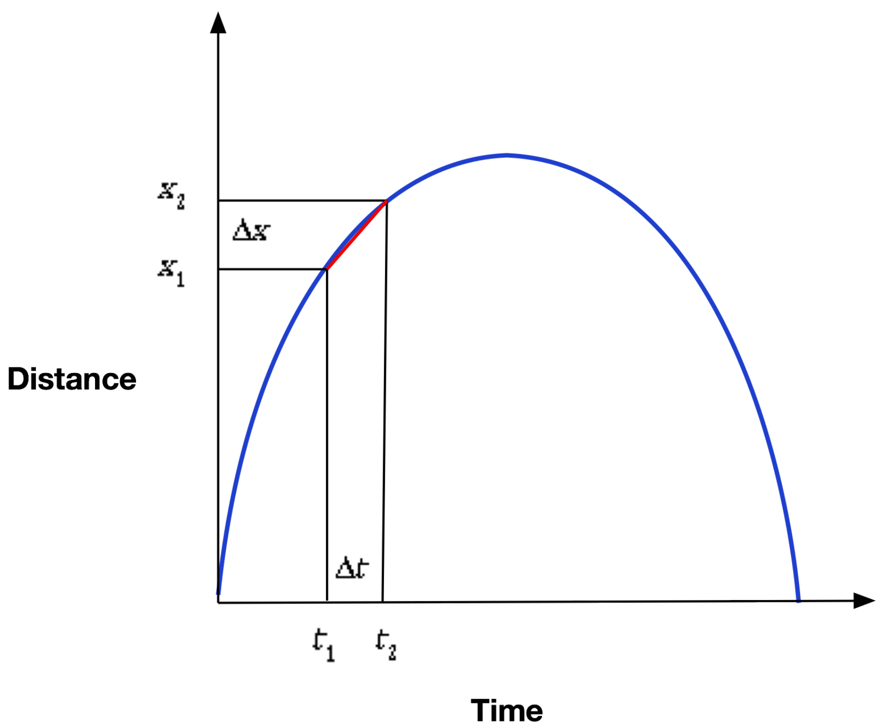
Now let us consider a scenario in which motion is not linear. If a projectile is thrown upwards and returns to earth, the position vs time graph will resemble the following;



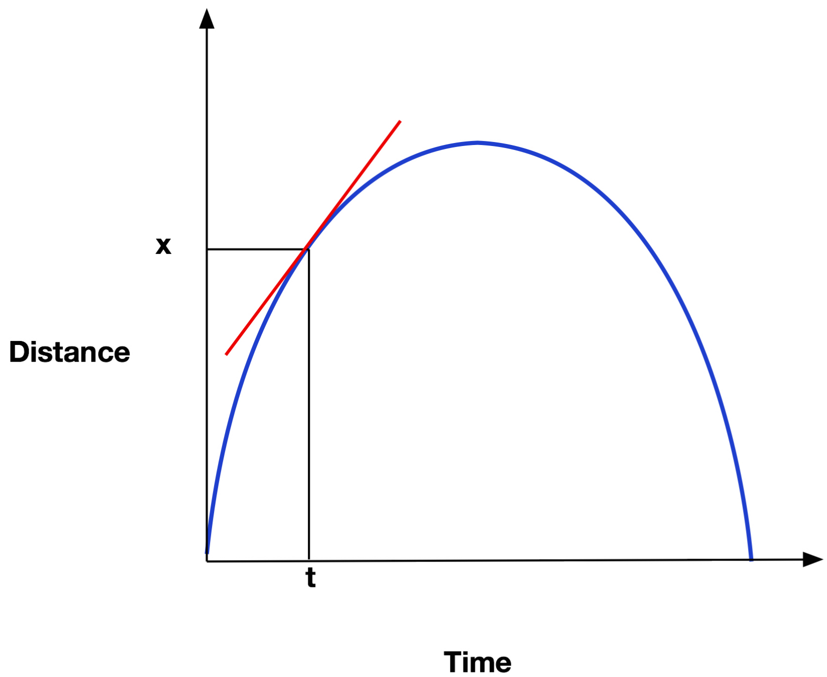
In establishing a velocity between two points in space (x_1 and x_2) corresponding to two points in time (t_1 and t_2) an estimate can be made by dividing the change in distance (Δx) by the change in time (Δt). In our case, the velocity is represented by the red line. The area between the blue line and the red line, however, represents an error. We have determined an **estimate** of velocity.

Let us consider what happens when the Δt and the Δx become smaller. The graph would take on the following shape;

You will note that the area between the blue line and the red line has become smaller. This would correspond to a smaller error than that in the first graph.



Let us now go one step further. What if the distance represented by Δt approached the limit. In essence, would resemble a line more so than a rectangle, and the area between the blue line and the red line would approach zero, thereby indicating an error approaching zero. (This is the basis of calculus.) Also, the red line would appear as the tangent of the blue line indicating an **instantaneous rate of change** at that point. See below.



Now let us review our original definitions of scalars and vectors. Scalars defined as quantities of measurement with no directional component and vectors described as quantities of measurement with a directional component. While the definitions appear simple, their application is somewhat more complicated.

Mathematical convention dictates placing an arrow over the variable in question to denote it as a vector quantity. For example, should the variable be denoted by the letter “x”, then

x = Scalar

$\vec{x}$ = Vector

As a review, a distinction must be drawn between scalar and vector quantities.

Distance = Scalar

Displacement = Vector

Speed = Scalar

Velocity = Vector

Acceleration = Vector

Vectors have a magnitude and direction, but the end points matter, not the motion in between the end points. In order to obtain the value for a vector at an instantaneous moment in time, the end points between the measurements must approach the limit of zero.